55. $W = PV + \frac{V\delta v^2}{2g} = a + \frac{b}{g}$ a, b constant

W = work done by the heart per unit time

g = acceleration of gravity

$dW = -\frac{b}{g^2} \cdot dg$

$$\frac{dW_{moon}}{dW_{earth}} = \frac{-\frac{\cancel{b}}{5.2^2}\cancel{dg}}{-\frac{\cancel{b}}{32^2}\cancel{dg}} = \frac{-\frac{1}{5.2^2}}{-\frac{1}{32^2}} = \frac{32^2}{5.2^2}$$

$\sim$

Nov 3-12:58 PM

46 estimate ΔA with dA

$\Delta A \approx dA$

$A = \pi r^2$

a) $dA = 2\pi r\, dr = 2\pi \cdot 2(.02) = .2513$

b) $\frac{.2513}{\pi \cdot 2^2} \times 100\% = \cancel{78.54\%}$ $.02 = 2\%$

% change in Area

% change in radius $\frac{.02}{2} = .01 = 1\%$

Nov 3-1:11 PM

4.6 related rates (related derivatives)

2 variables change over time & are related

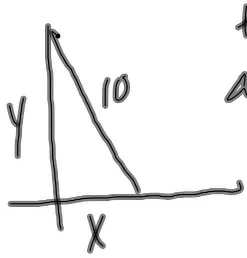
given: how fast one of the variables changes (derivative)

find: how fast the other variable changes (derivative)

so we need an equation that relates the derivatives

Nov 3-1:16 PM

ex. 10 ft ladder leans against a wall. the bottom of the ladder is pulled away at a rate of 2 ft/sec. Does the top fall at a steady rate? no



$\frac{dx}{dt} = 2\ \frac{ft}{sec}$ find $\frac{dy}{dt}$

$x^2 + y^2 = 10^2$ take derivative with respect to t

$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0$ solve for $\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{-2x}{y}$ $\frac{dy}{dt}$ increases with time

Nov 3-1:21 PM

How fast is the top falling when the bottom is 4 ft from the wall?

$x^2+y^2=10^2$ $x=4$ $y=\sqrt{10^2-4^2}$

$$\frac{dy}{dt}=\frac{-2(4)}{\sqrt{100-16}}\approx -.8729\ \frac{ft}{sec}$$

going down

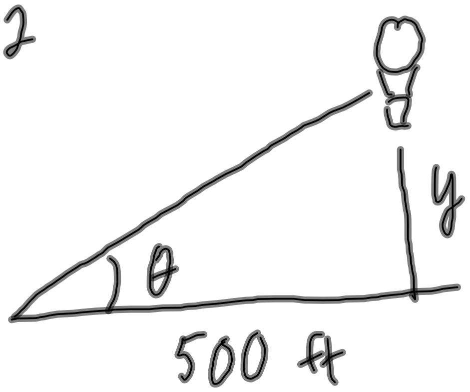
y decreasing

$$\frac{dy}{dt}=\frac{-2x}{y}$$

Nov 3-1:34 PM

Ex 2

p247



find $\frac{dy}{dt}$

given $\frac{d\theta}{dt}=.14\frac{rad}{min}$

$$\tan\theta=\frac{y}{500}$$

$y=500\tan\theta$ take der w.r.t. t

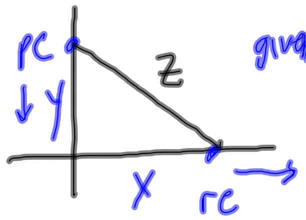
$$\frac{dy}{dt}=500\sec^2\theta\,\frac{d\theta}{dt}$$

$$=500\sec^2\left(\frac{\pi}{4}\right)(.14)$$

$$=140\ \frac{ft}{min}$$

Nov 3-1:37 PM

Ex 3



given: $\frac{dz}{dt} = 20_{mph}$ $\frac{dy}{dt} = 60_{mph}$

find: $\frac{dx}{dt}$

$y = .6$
$x = .8$

$z^2 = x^2 + y^2$ diff wrt t

$\cancel{2}z\frac{dz}{dt} = \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}$ subst.

$\sqrt{.6^2 + .8^2} \cdot 20 = .8\frac{dx}{dt} + .6 \cdot 60$ solve

$\frac{dx}{dt} = \frac{20 - .6(60)}{.8} = 70 \text{ mph}$

Nov 3-1:48 PM